Rev Quant Finan Acc (2008) 31:55–70
DOI 10.1007/s11156-007-0061-8

ORIGINAL PAPER

Analysing the performance of managed funds using the wavelet multiscaling method

Francis In · Sangbae Kim · Vijaya Marisetty · Robert Faff

Published online: 13 October 2007

Abstract We propose a new approach for investigating the performance of managed funds using wavelet analysis and apply it to an Australian dataset. This method, applied to a multihorizon Sharpe ratio, shows that the wavelet variance at the short scale is higher than that of the longer scale, implying that an investor with a short investment horizon has to respond to every fluctuation in the realized returns, while for an investor with a much longer horizon, the long-run risk associated with unknown expected returns is not as important as the short-run risk. Using multihorizon Sharpe ratios of six groups of managed funds, we find that none of the fund groups are dominant over all time scales.

Keywords Performance measure · Wavelet analysis · Sharpe ratio · Australian managed funds

JEL Classifications G23 · G21 · G10

1 Introduction

The Sharpe ratio is an extensively used performance evaluation measure by practitioners in a mean-variance framework (Sharpe 1966, 1994). Due to its wide practical popularity,

F. In · V. Marisetty · R. Faff (✉)
Department of Accounting and Finance, Monash University, Victoria 3800, Australia
e-mail: Robert.Faff@buseco.monash.edu.au

F. In
e-mail: Francis.In@buseco.monash.edu.au

V. Marisetty
e-mail: Vijaya.Marisetty@buseco.monash.edu.au

S. Kim
School of Business Administration, Kyungpook National University, 1370 Sankyuk-dong, Puk-ku, Daegu 702-701, Korea
e-mail: sbkim@mail.knu.ac.kr

there has been ongoing research to enhance the Sharpe ratio and improve/robustify its application. For example, Modigliani and Modigliani (1997) created a variant they called the "risk-adjusted performance" measure which scales a fund's return by the ratio of the market risk to fund risk. In other words the "M^2" metric asks the question: if the fund portfolio's risk had been the same as the market portfolio, what returns would the fund have achieved?

Other papers have sought to explore the statistical and inferential properties of the Sharpe ratio. A time-honoured example of this is Jobson and Korkie (1981), in which (among other things) the low power of such testing was highlighted. More recently, others have followed in this vein—see for example, Owen and Rabinovitch (1999). Recently we have Lo (2002) who derives explicit distributional properties of the Sharpe ratio under different scenarios.[1] More recently, Knight and Satchell (2005) investigated the exact properties of the Sharpe ratio when prices are log-normal and developed alternative unbiased estimators, depending on the definition of returns employed. Two further examples of different strands from the literature focused on the Sharpe ratio are: Dowd (1999) who is concerned with building an integrated framework of financial risk management based on the "Sharpe rule" and Ziemba (2005) who develops a symmetric downside-risk Sharpe ratio.

Nielson and Vassalou (2004) represents another recent and very relevant example from this literature that seeks to enhance the usefulness of Sharpe ratios. Specifically, they provide a continuous time version of the Sharpe ratio and prove that the one-period variant commonly used by practitioners fails to adequately capture the changing volatility of the return series. They show that the continuous time version is equal to the ordinary Sharpe ratio plus half of the volatility of the fund.

As indicated in Levy (1972), the Sharpe ratio is sensitive to the investment horizon. The holding period that is relevant for portfolio allocation is the length of time investors hold any stocks or bonds, no matter how many changes are made among the individual issues in their portfolio (Siegel 1998, p. 29). Consider an investment company with a large number of investors and money managers. Clearly, the investors and the money managers make decisions over different time scales. Suppose, for simplicity, that the investment horizon of an investor is one year and that the investment company reviews the performance of the money manager every quarter, using the Sharpe ratio. The money manager will therefore focus on the three-month performance of a portfolio, while the investor will concentrate on the one-year performance. Thus, for this investor, information provided by the money manager may not be relevant. Indeed, sometimes it could even be misleading.

To provide the best service for diversified investors, the Sharpe ratio needs to be constructed over different investment horizons (Kim and In 2005a). In other words, the horizon sensitivity of the Sharpe ratio is a very important issue in evaluating the performance of one or more portfolios. An investor might not be interested in short-term performance of portfolios. Indeed, institutional investors such as pension funds have a very long investment horizon. A common practice in measuring the n-period Sharpe ratio is to use the expected return and standard deviation of the one-period investment by taking the corresponding scaling quantity into account (see Levy 1972). This n-period Sharpe ratio is

[1] Lo's (2002) efforts have however, come under close scrutiny regarding the generalizability of his analysis. For example, Wolf (2003) highlights that the main distributional result for the Sharpe ratio in Lo (2002) critically assumes normality of returns. As such, the confidence intervals circumstances will be biased in the direction of being too narrow. In response, Lo (2003) clarifies the more general result for the more realistic setting of non-normal distributions.

a complex, nonlinear function of the expected return and standard deviation of the one-period investment. This type of scaling is only valid if the underlying data is identically and independently distributed (Hodges et al. 1997), which may not be the case for certain financial time series.

We propose a new approach for investigating the performance of managed funds using a wavelet analysis of the multihorizon Sharpe ratio (Kim and In 2005a).[2] Our study extends the current literature in two important ways. First, to the best of our knowledge, the current paper is the very *first* to investigate the performance of managed funds, using wavelet analysis. As such, this study helps to deepen our understanding of the true relationship between risk and return of managed funds over different time scales. The results therefore should be of interest to both investors and fund managers.

Second, a key premise underlying the research in this paper is that the relationship between financial variables may be better expressed in terms of restrictions to given time scales. From the empirical analysis of a sample of Australian managed funds, we find that the wavelet variance at the short scale is higher than that of the longer scale, implying that an investor with a short investment horizon has to respond to every fluctuation in the realized returns, while for an investor with a much longer horizon, the long-run risk associated with unknown expected returns is not as important as the short-run risk. Using the multihorizon Sharpe ratios for six different groups of managed funds, we find that none of the fund groupings are dominant over all time scales, indicating that evaluating the performance of the managed funds depends on the investment horizon.

The remainder of the article is organized as follows: Section 2 presents some institutional features of the Australian managed funds sector and also a review of the Australian fund performance literature. In Sect. 3, we discuss how to derive the multihorizon Sharpe ratio using wavelet analysis. Data and the empirical results are discussed in Sect 4. In Sect. 5, summary and concluding remarks are presented. Finally, a technical appendix presents the econometric methodology for wavelet analysis.

2 Australian managed funds sector

The Australian managed funds sector, with more than US \$430 billion dollars of assets, is the fourth largest managed funds sector in the world.[3] However, research on the performance of Australian managed funds is relatively sparse. The sector is broadly divided into two categories, namely, wholesale and retail funds. The division is based on the nature of subscription into managed funds. In the case of wholesale funds, investors contribute indirectly to the funds. The trustees of the managed fund take the decision of investment on behalf of the investors. Thus, wholesale funds are pooled investments. Retail funds on the other hand receive money directly from the investors. Due to the pooled nature of investment, wholesale funds are larger than retail funds and incur lower operating costs. This is also reflected in the fees charged by these funds. The management expense ratio (MER) of wholesale funds is generally lower than retail funds. The product lines of managed funds includes superannuation, non-superannuation and life insurance products.

[2] In the recent times there have been a few new approaches to measure managed funds performance. For instance, Chang et al. (2003) measure hedging timing ability of fund managers along with their market timing and security selection skills.

[3] Investment and Financial Services Association Limited (IFSA, Australia).

Australian fund performance studies by and large are replications of US studies. Bird et al. (1983) found that only one out of 27 managers showed some evidence of superior performance through security selection skills. Sinclair (1990) using Treynor and Mazuy (1966) and Henriksson and Merton (1981) models on 16 pooled superannuation funds for 83 monthly observations found evidence of funds exhibiting positive alphas. However, Hallahan and Faff (1999) documented little evidence of successful market timing of Australian equity trust managers. Marisetty and Ariff (2003) using the Lee and Rahman (1990) measure, found no evidence of market timings skills for a sample of 90 retail funds. Benson and Faff (2003) tested the performance of 70 Australian international equity trusts. They found that international equity trusts during 1990–1999 exhibited negative timing and selection skills. Sawicki and Ong (2003) applying the Ferson and Schadt (1996) methodology on 97 wholesale Australian superannuation funds, from 1983–1995, found that the negative market timing reported in earlier studies will disappear after controlling for changing market conditions, similar to Ferson and Schadt (1996). Gallagher (2001) evaluated the market timing and selection skills of 33 wholesale superannuation fund managers from 1992–1998 and found no evidence of superior timing and selection skills. Holmes and Faff (2004) found negative performance for Australian multi-sector funds. In summary, these results are broadly consistent with US based studies.

3 Multihorizon Sharpe ratio using wavelet analysis[4]

3.1 Sharpe ratio basics

Although over the years much academic work has been devoted to performance measurement, practitioners typically use two popular performance measures: the Sharpe ratio and/or Jensen's alpha. It is well known that the traditional Sharpe ratio is the appropriate measure of performance of a portfolio if the distribution of its rate of return is normally distributed and Sharpe (1994) shows the fundamental case for using the Sharpe ratio. Suppose we have a portfolio, i, with a return, R_i and a risk-free asset, denoted by f, with return R_f. The Sharpe ratio (SR for portfolio i) can be defined by:

$$SR_i = \frac{\bar{R}_i - \bar{R}_f}{\sigma_i} \tag{1}$$

where $\bar{R}_i$ is the mean return of the portfolio, $\bar{R}_f$ is the mean return on the risk-free asset and σ_i is the standard deviation of the portfolio return.

Assuming all asset returns to be normally distributed, the CAPM tells us that in equilibrium the highest attainable Sharpe ratio is that of the market index. A ratio higher than that therefore indicates superior performance. However, many financial time series do not follow the normal distribution. Possible measures of performance when returns are not distributed normally are discussed in the previous literature (see for example, Leland 1999, Stutzer 2000 and Kazemi et al. 2003). In particular, Kazemi et al. (2003) derive the adjusted Sharpe ratio by transforming the distribution of the portfolio returns so that its distribution will match that of a benchmark. This approach is useful because it does not require the normality of the returns.

[4] This section draws heavily on Kim and In (2005a).

Suppose that we have a portfolio, i, with a return, R_i, and a benchmark, B, with a return, R_B. The adjusted Sharpe ratio (ASR) for the portfolio, i, can be defined as follows:

$$ASR_i = \frac{\bar{R}_B - \bar{R}_f}{\sigma_B} + \frac{(1 + \bar{R}_f)(1 - P)}{\sigma_B} \tag{2}$$

where

$$P = \frac{E[F(1 + R_i)]}{1 + R_f} \tag{3}$$

where $F(1 + R_i)$ indicates a function that matches the distribution of the portfolio returns with that of the benchmark returns; $E(\cdot)$ means expectation under the risk-neutral probability distribution; and P is the current value of $F(1 + R_i)$, respectively.[5] Note that the Sharpe ratio of the benchmark is the same as the Sharpe ratio, Eq. 1. In other words, the difference between the Sharpe ratio of the benchmark and the adjusted Sharpe ratio of the portfolio is the term $(1 + \bar{R}_f)(1 - P)/\sigma_B$. In the adjusted Sharpe ratio, the value of P plays a crucial role in comparing the performance of a portfolio. Specifically, if $P < 1 (P > 1)[P = 1]$, the portfolio is a better (inferior) [equally performing] investment than (to) [with] the benchmark.

3.2 Wavelet adjusted Sharpe ratio

Financial security markets are complex systems which comprise a wide range of investors with different time horizons, thereby impacting their investment decision-making processes. As a consequence, observed returns as signals can be examined across different resolution levels, to deduce information regarding investors' time horizons. In this context, wavelet analysis is a natural tool for investigating portfolio allocation decisions, as wavelets allow us to decompose a signal into multiresolution components: namely, 'fine' and 'coarse' resolution components.

By design, the wavelet's strength rests on its ability to simultaneously localize a process in time and scale. At high scales, the wavelet has a small centralized time support enabling it to focus on short-lived time phenomena such as jumps. At the other end of the spectrum, low scales provide the wavelet with a large time support allowing it to identify long-term periodic behaviour. By moving from low to high scales, the wavelet zooms in on the behaviour of a process at a particular point in time, identifying 'micro' features such as

[5] For more details of the derivation of the adjusted Sharpe ratio and its algorithm, see Kazemi et al. (2003). To calculate the adjusted Sharpe ratio, it is important to match the distribution of a portfolio to that of benchmark. To do so, the first step is to estimate an approximation of function $F(1 + R_i)$. Kazemi et al. (2003) show that after transforming the return of a portfolio, its distribution is almost same as that of benchmark. Based on the function $F(1 + R_i)$, we estimate the value of P. In our paper the calculated values of P are subject to sampling variation. For this reason, we calculate the statistical inference using bootstrap methods by generating 3,000 random returns from the risk neutral distribution. The average across replications is very similar to the original estimates of the value of P, which means that no correction for small-sample bias is needed (we do not present the results but they are available on request).

singularities, jumps, and cusps. Alternatively, the wavelet can zoom out to reveal the long, smooth features of a series (Nielson and Frederiksen in press).[6]

Given the adjusted Sharpe ratio, the multihorizon Sharpe ratio can be constructed using the wavelet variance and local average at scale λ_j as follows:

$$ASR_i^w(\lambda_j) = SR_B^w(\lambda_j) + \frac{(1 + \bar{R}_f(\lambda_j))(1 - P(\lambda_j))}{\sqrt{\tilde{v}_B^2(\lambda_j)}} \tag{4}$$

$$\text{where} \quad SR_B^w(\lambda_j) = \frac{\bar{R}_B(\lambda_j) - \bar{R}_f(\lambda_j)}{\sqrt{\tilde{v}_B^2(\lambda_j)}} \tag{5}$$

where $\bar{R}_B(\lambda_j)$ and $\bar{R}_f(\lambda_j)$ are the mean values of the benchmark portfolio return and the risk-free rate at scale λ_j. These mean values are calculated using the scaling coefficients, following Gençay et al. (2003). In this specification, SR_B^w and ASR_i^w indicate the wavelet multiscale Sharpe ratio of the benchmark portfolio and the adjusted wavelet multiscale Sharpe ratio of a portfolio, which can be varying depending on the wavelet scales (i.e., investment horizons). The null hypothesis is: H_0: $P(\lambda_j) > 1$ (i.e. no out-performance of the portfolio at time scale λ_j.) versus the alternative hypothesis of interest, H_1: $P(\lambda_j) < 1$, (namely that the portfolio outperforms the benchmark at time scale λ_j). Thus, the Sharpe ratios at different scales represent the performance measure of a portfolio at various frequencies (in other words, at various time scales). Moreover, the multihorizon Sharpe ratio has an ability to capture performance of a portfolio at various frequencies, compared to the traditional Sharpe ratio.

4 Data and empirical results

4.1 Data

Managed funds in Australia can be broadly classified as wholesale and retail funds based on the nature of contribution. If the contribution to the fund is made directly by the investor with voluntary choice they are termed retail funds. In the case of wholesale funds contribution to the fund is by the employer of the investor. The data employed are 36 wholesale managed fund monthly returns and 59 retail managed fund monthly returns obtained from the ASSIRT database from November 1990 to December 2001. Our sample includes all funds that exist during the last 15 years. Our sample clearly has survivorship bias. Such bias is unavoidable in our study given the very nature of our analysis—the multi-scaling approach we use requires both short and long horizon fund returns. While survivorship is a weakness of our study, it is a necessary ‘evil’ in order to make the long horizon part of the analysis sensibly viable.

We choose to perform our analysis according to size groupings, given the extensively documented evidence that size is a key characteristic of managed funds. To construct the large, medium and small wholesale fund returns, we first sort the wholesale fund returns by their size. Using this size ranking, the largest 12 wholesale funds are allocated into the “large” managed funds group, whose returns are calculated by an equally weighted average. The medium and small wholesale fund returns are calculated using the same

[6] More detailed explanation of wavelet analysis and the technical details of how we implement wavelet decomposition, is provided in the technical appendix.

approach. Similarly, the large retail fund returns are calculated by an equally weighted average of the largest 19 retail funds. Thus, we have six return series representing six different size based groups. To calculate the excess return, the risk-free returns are required. In this analysis, we use the one-month deposit rate obtained from Datastream.[7]

Table 1 presents several summary statistics for the monthly data of six fund group returns (large, medium, and small wholesale funds, large, medium, and small retail funds) and risk-free returns. As shown in Panel A of Table 1, the mean values are all positive and close to zero. Nevertheless, these figures show that the small fund returns (wholesale and retail) are higher than the counterpart large and medium fund returns. Another noticeable observation is that variance of the small fund returns are higher than those of the large and medium fund returns, implying that the higher the return, the higher the risk. Also it is noted that over the sample period wholesale funds historical average returns are slightly higher than retail funds.

The measures for skewness, kurtosis and Jarque-Bera statistics are also reported to check whether monthly data are normally distributed. The sign of skewness varies; however, the magnitudes depend on the particular security. The Jarque-Bera statistics also indicate that all data are not normally distributed. The LB(15) for the squared return series is highly significant for all assets, suggesting the possibility of the presence of autoregressive conditional heteroskedasticity.[8] Notice that due to the non-normality of all returns, we use the adjusted Sharpe ratio instead of the traditional Sharpe ratio.

We report the unconditional contemporaneous correlation coefficients among the risk-free return and the six Australian managed funds in Panel B of Table 1. The correlation ranges from 0.816 between the small wholesale fund and the small retail fund to 0.988 between the large wholesale fund and the medium wholesale fund. Overall, it is observed that the correlation coefficients between the Australian managed funds are very high.

4.2 Empirical results

The purpose of this article is to examine the performance of the sample of Australian managed funds. To do so, we use the multihorizon Sharpe ratio using wavelet analysis and we decompose our data up to scale 5.[9] Since we use monthly data, scale 1 represents 2–4 month period dynamics. Equivalently, scale 2, 3, 4, and 5 correspond to 4–8, 8–16, 16–32, and 32–64 month period dynamics, respectively.[10]

First, we examine the variances of the six returns against various time scales. An important characteristic of the wavelet transform is its ability to decompose (analyze) the variance of the stochastic process. Figure 1 illustrates the wavelet variance of the six series against the wavelet scales.[11] In this figure, instead of presenting the exact number of the

[7] The Australian 13-week Treasury notes were not used for the risk-free rate since they were discontinued in August 2000.

[8] LB(15) is the Ljung-Box statistic for up to 15 lags, distributed as $\chi 2$ with 15 degrees of freedom.

[9] Considering the balance between the sample size and the length of the wavelet filter, we settle with the Daubechies extremal phase wavelet filter of length 4 (D(4)). For the various wavelet filters, see Gençay et al. (2002, pp. 114–115).

[10] According to Gençay et al. (2003), the spectrum of raw monthly return series contains all frequencies between zero and 1/2 cycles, equivalent to the 0 to 2-month period in our data frequency.

[11] For statistical inference, we also calculate the confidence interval for six series. However, for the sake of a clear presentation we do not plot the confidence intervals.

Table 1 Basic statistics of Australian retail and wholesale superannuation fund monthly returns during 1990–2001

	Wholesale funds				Retail funds		
	Risk-free	Large	Medium	Small	Large	Medium	Small
Panel A. Descriptive statistics							
Mean	0.005	0.253	0.245	0.336	0.158	0.140	0.169
Variance	0.000	2.393	2.188	4.714	1.803	1.820	2.136
Skewness	1.424	–0.448	–0.358	–0.231	–0.514	–0.497	–0.568
Kurtosis	1.941	–0.127	–0.312	1.235	0.303	–0.069	–0.034
JB	66.298 (0.000)	4.567 (0.102)	3.412 (0.182)	9.705 (0.008)	6.412 (0.041)	5.548 (0.062)	7.211 (0.027)
LB(15) for x	1223.563 (0.000)	23.834 (0.021)	21.980 (0.038)	22.411 (0.033)	29.179 (0.004)	22.099 (0.036)	26.023 (0.011)
LB(15) for x^2	1141.046 (0.000)	11.095 (0.521)	15.326 (0.224)	17.388 (0.136)	17.428 (0.134)	8.967 (0.706)	7.741 (0.805)

	Risk-free	Large wholesale funds	Medium wholesale funds	Small wholesale funds	Large retail funds	Medium retail funds	Small retail funds
Panel B. Correlation Matrix							
Risk-free	1.000	0.023	0.051	0.040	0.037	0.064	0.035
Large wholesale funds		1.000	0.988	0.828	0.966	0.979	0.955
Medium wholesale funds			1.000	0.837	0.956	0.974	0.953
Small wholesale funds				1.000	0.825	0.820	0.816
Large retail funds					1.000	0.978	0.958
Medium retail fund						1.000	0.968
Small retail funds							1.000

Note: the monthly returns for the risk-free rate, six fund group range from November 1990 to December 2001. Significance levels are in parentheses. LB(n) is the Ljung-Box statistic for up to n lags, distributed as χ^2 with n degrees of freedom. JB indicates the Jarque-Bera statistic. Skewness and kurtosis are defined as $E[(R_t-\mu)]^3$ and $E[(R_t-\mu)\ ]^4$, respectively, where μ is the sample mean

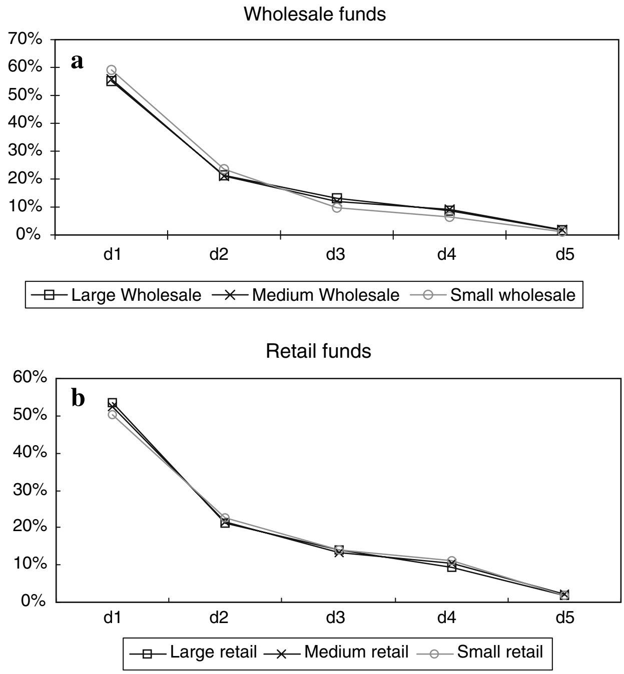

Fig. 1 Estimated wavelet variances. (**a**) Wholesale funds. (**b**) Retail funds. *Note*: The *y*-axis indicates the wavelet variance and the *x*-axis indicates the wavelet time scale. To calculate the wavelet variance of each portfolio, we decompose each time series up to level 5, using the Daubechies extremal phase wavelet filter of length 4 (D(4)). d1, d2, d3, d4, and d5 represent 2–4, 4–8, 8–16, 16–32, and 32–64 month period dynamics, respectively

wavelet variances, the specific contributions to sample variance of each scale are presented. Percival and Walden (2000) and In and Kim (2006) show that a wavelet variance in a particular time scale indicates the contribution to sample variance. For example, the sample variances are 0.253 for the large wholesale managed funds, 0.245 for the medium wholesale funds, and 0.336 for the small wholesale funds, respectively, and 55.08%,[12] 55.81%, and 59.09% of the total variances of the three fund returns, respectively, are accounted for by the lowest scale (scale 1). This result implies that an investor with a short investment horizon has to respond to every fluctuation in the realized returns, while for an investor with a much longer horizon, the long-run risk is considerably less. The retail fund returns show a similar pattern to the wholesale fund returns.

It is of interest to compare the annualized wavelet variances (risks) over the different time scales. In financial valuation models, it is commonly required to calculate an annualized risk coefficient. Under the random walk model, the risk of an asset at any time scale is estimated by the linear rescaling of the risk from other time scales. Conventionally, the risk is scaled at the square root of time. Our results are presented in Fig. 2. As observed in the figure, the annualized variances decrease as the time scale increases, with a smoother decreasing pattern. This result implies that an investor with a short investment horizon has to respond to every fluctuation in the realized returns, while for an investor with a much

[12] These figures are calculated by the normalization of wavelet variance using the sample variance. For more details, see In and Kim (2006).

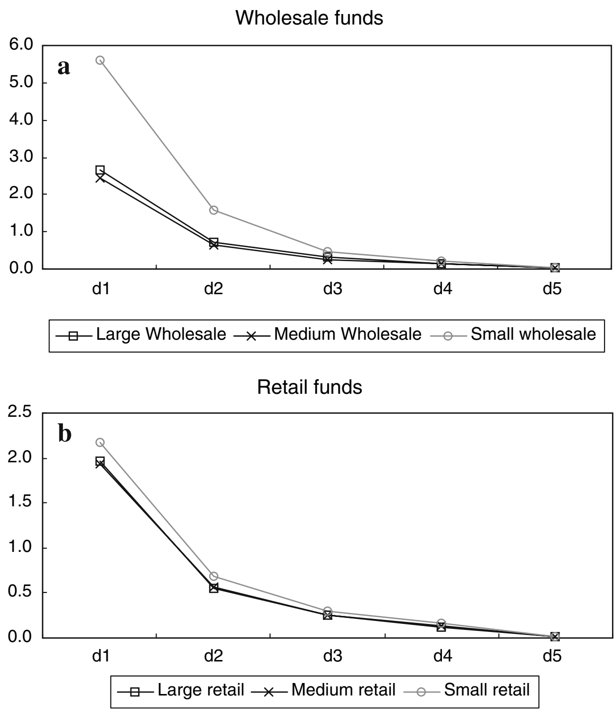

Fig. 2 Estimated annualized wavelet variances. (**a**) Wholesale funds. (**b**) Retail Funds. *Note*: The *y*-axis indicates the wavelet variance and the *x*-axis indicates the wavelet time scale. To calculate the wavelet variance of each portfolio, we decompose each time series up to level 5, using the Daubechies extremal phase wavelet filter of length 4 (D(4)). d1, d2, d3, d4, and d5 represent 2–4, 4–8, 8–16, 16–32, and 32–64 month period dynamics, respectively

longer horizon, the long-run risk is significantly less. Overall, the returns on small fund returns are more volatile than those of the large and medium fund returns for all wavelet scales, while the annualised wavelet variances for the large and medium funds do not show much difference.

Turning to our main purpose, the multihorizon Sharpe ratios, using the large wholesale fund returns as a benchmark, are presented in Table 2.[13] Table 2 shows that the Sharpe ratio for each fund return increases as the holding period is extended, except for scale 2 (d2). This result is consistent with Lo (2002). The Sharpe ratio for the large wholesale managed fund returns is 0.26% for the first wavelet scale, equivalent to a 2–4 month period, and increases to 38.01% at the longest wavelet scale, equivalent to a 32–64 month period. A similar pattern is observed in the other asset returns. For example, the Sharpe ratio of the small retail fund starts at 0.06% at the first wavelet scale, and rises to 35.50% at the longest wavelet scale. This result simply indicates that the Sharpe ratio is not time consistent. In other words, the trading strategy that leads to the most desirable portfolio for a 2–4 month period (scale 1) and for four consecutive 2–4 month periods is not the same as the strategy that gives the maximum Sharpe ratio for an 8–16 month period (scale 3), suggesting that when the Sharpe ratio is used as a performance measure, the horizon effect

[13] We choose to benchmark against the large wholesale funds because this achieves the twin objective of having a 'market like' benchmark, while allowing a direct examination of the impact of fund size.

Table 2 Multihorizon Sharpe Ratio for Australian Retail and Wholesale Superannuation Funds

	Wholesale funds(%)			Retail funds(%)		
	Large	Medium	Small	Large	Medium	Small
Raw data	16.03	16.23	15.71	11.36	10.15	11.41
d1	0.26	0.10	0.08	0.22	0.27	0.06
d2	–0.51	–0.97	–0.82	–0.60	–0.69	–0.68
d3	0.57	0.60	1.08	0.71	0.62	0.74
d4	8.89	9.25	8.10	7.27	7.43	7.01
d5	38.01	34.99	41.28	41.33	48.35	35.50

Note: The large wholesale fund is considered as a benchmark. The multihorizon Sharpe ratios of the other managed fund returns are calculated using Eq 4. To calculate the multihorizon Sharpe ratio of each fund return at scale λ_j, we decompose each time series up to level 5, using the Daubechies extremal phase wavelet filter of length 4 (D(4)). d1, d2, d3, d4, and d5 represent 2–4, 4–8, 8–16, 16–32, and 32–64 month period dynamics, respectively

should be taken into account. Overall, our results while consistent with those of Kim and In (2005a), are different from those of Hodges et al. (1997). The latter researchers found that the Sharpe ratio eventually declines in all cases and that bonds become more attractive than stocks for long holding periods.

Table 2 presents the relative rankings of the six fund groups over the wavelet scales, as well as for the raw data. It is shown that that no single fund grouping dominates all others over all time scales. For example, in the d1 case large (medium) wholesale (retail) are ranked best. However, for d3 small funds are best, while for d5 small funds are best amongst the wholesale groups only. These results imply that the performance of managed funds critically depend on the investment horizon.

4.3 Summary and concluding remarks

Managed fund performance has long been of interest to financial economists, mainly because of its implications for investors. This paper adopts a new approach to using the multihorizon Sharpe ratio to evaluate the performance of six different groups of Australian managed funds over various time scales. Although the Sharpe ratio has become an important part of modern financial analysis, its applications have not been accompanied by examination of the different investment horizons, which is an important factor for investors.[14] In this article, we examine the multihorizon Sharpe ratio using wavelet analysis. Wavelet analysis has the advantage of being able to decompose the time series over various time scales. This advantage allows us to investigate the behavior of our data over multiple horizons. This new approach is based on a wavelet multiscaling method that decomposes a

[14] The logic is as follows. Financial managed fund markets are heterogeneous and made of investors and traders with different investment time horizons. Consider the large number of investors who trade in the security market and make decisions over different time scales. One can visualize traders operating minute-by-minute, hour-by-hour, day-by-day, month-by-month, and year-by-year. In fact, due to the different decision-making time scales among traders, the true dynamic structure of the relationship between the performance of the managed funds and risk factors will *vary* over different time scales associated with those different horizons. Therefore, it is imperative to know that the true performances of the different managed funds will only be revealed when the Sharpe ratio is decomposed by the *different time scales* or *different investment horizons*.

given time series on a scale-by-scale basis. Wavelet analysis has the advantage of being able to decompose the time series over the various time scales. This advantage allows us to investigate the behavior of our data over multiple horizons.

Our empirical results indicate that the wavelet variances of both the stock and bond markets decrease as the wavelet scale increases, implying that an investor with a short investment horizon has to respond to every fluctuation in the realized returns, while for an investor with a much longer horizon, the long-run risk associated with unknown expected returns is not as significant as the short-run risk. In addition, the Sharpe ratio at the long scale is much higher than that at the short scale, implying that the Sharpe ratio is *not* time consistent. This result is consistently confirmed by wavelet analysis. Since risk and value (performance) are timescale-dependent concepts, any attempt to measure performance, such as a Sharpe ratio, must take into account the investment horizon effect. Wavelets are natural tools for this purpose.

Acknowledgments The authors thank an anonymous referee for helpful comments and are pleased to acknowledge the support of the Australian Research Council, grant number DP0557172.

Technical appendix wavelet analysis

Wavelet analysis refers to the representation of a signal in terms of a finite length or fast decaying oscillating waveform (known as the mother wavelet). Financial markets are comprised of investors and traders with different investment time horizons—there is considerable heterogeneity of investment time horizon. At the heart of most trading mechanisms are the market makers. At the next level up are the intraday investors who carry out trades only within a given trading day. Then there are day investors who may carry positions overnight, and then short-term traders and finally at the top of the tree are long-term traders. Overall, it is the aggregation of the activities of all investors for all different investment horizons that ultimately generates market prices. Therefore, market activity is heterogenous with each investment horizon dynamically providing feedback across the different time scales (Dacorogna et al. 2001). The implication of a heterogenous market is that the true dynamic relationship between the various aspects of market activity will only be revealed when the market prices are decomposed by the different time scales or different investment horizons. The main advantage of wavelet analysis is the ability to decompose a time series, measured at the highest possible frequency, into several time scales.[15]

A major innovation of our paper is the introduction of a new approach—the wavelet multiscaling method—for investigating the multihorizon Sharpe ratio. We achieve this through an application to the performance of a sample of Australian managed funds. In this section, we briefly review how to derive the wavelet coefficients and how to derive wavelet variance over various time scales.

One useful way to think about wavelets is to consider wavelets and scaling filters.[16] The relationship between filter banks and wavelets is extensively discussed in Strang and

[15] Wavelet analysis is relatively new in economics and finance, although the literature on wavelets is growing rapidly. Applications in these fields include decomposition of economic relationships of expenditure and income (Ramsey and Lampart 1998a, b), systematic risk in the capital asset pricing model (Gençay et al. 2003), the multiscale relationship between stock returns and inflation (Kim and In 2005b), and a multiscale hedge ratio (In and Kim 2000).

[16] According to Ramsey (2002), one can approach the analysis of the properties of wavelets either through wavelets or through the properties of the filter banks.

Nguyen (1996) and Percival and Walden (2000). Like conventional Fourier analysis, wavelet analysis involves the projection of a signal onto an orthogonal set of components—sine and cosine functions in the case of Fourier analysis and wavelets in the case of wavelet analysis. Wavelet analysis enables us to decompose the data into several time scales. To show the multiscale decomposition of a portfolio return series by wavelet analysis briefly, we employ the simple Haar wavelet, which is the first[17] wavelet filter.[18] The Haar wavelet filter coefficient vector, of length $L = 2$, is given by $h = (h_0, h_1) = (1/\sqrt{2}, -1/\sqrt{2})$. The complementary filter to h is the Haar scaling filter $g = (g_0, g_1) = (1/\sqrt{2}, 1/\sqrt{2})$. These filters possess the following attributes:

$$\sum_l h_l = 0, \quad \sum_l h_l^2 = 1, \quad \text{and} \quad \sum_l h_l h_{l+2n} = 0 \quad \text{for all integers} \quad n \neq 0. \tag{A.1}$$

$$\sum_l g_l = \sqrt{2}, \quad \sum_l g_l^2 = 1, \quad \text{and} \quad \sum_l g_l g_{l+2n} = 0 \quad \text{for all integers} \quad n \neq 0. \tag{A.2}$$

Equation A.1 indicates the basic three properties of the wavelet filter, namely, it: (1) sums to zero, (2) has unit energy, and (3) is orthogonal to its even shifts. The scaling filter (Eq A.2) follows the same orthonormality properties of the wavelet filter, namely, having unit energy and orthogonality to even shifts, but instead of differencing consecutive blocks of observations, the scaling filter averages them. Thus, g may be viewed as a local averaging operator.

Using the Haar wavelet filter coefficients h, when applied to a return series R_t, the wavelet coefficients can be obtained by:

$$\sqrt{2} d_{1t} = h_0 R_t + h_1 R_{t-1}, t = 0, 1, \ldots, T - 1. \tag{A.3}$$

The factor of $\sqrt{2}$ is required to ensure that the squared norm of the wavelet coefficients is equivalent to the squared norm of the return series. From this equation, it is observed that the wavelet coefficient d_{1t} is a weighted difference between consecutive returns. In a similar manner, the scaling coefficients can be obtained using the Haar scaling filter coefficient vector g as follows:

$$\sqrt{2} s_{1t} = g_0 R_t + g_1 R_{t-1}, t = 0, 1, \ldots, T - 1. \tag{A.4}$$

In contrast to d_1, the scaling coefficients s_1 are based on local averages (of length two) of the original returns. Collecting both sets of coefficients into $w = (d_1, s_1)$ yields the high-frequency and low-frequency content from the original returns. To derive the higher scale wavelet and scaling coefficients, it is necessary to calculate the higher scale wavelet and scaling filter coefficients using the first scale wavelet and scaling filters, h and g. To derive the scale 2 wavelet and scaling filters, let us define a filter $h' = (h_0, 0, h_1) = (1/\sqrt{2}, 0, -1/\sqrt{2})$ to be the Haar wavelet filter with a zero between the two coefficients. The scale 2 Haar wavelet filter is calculated by:

[17] In this context, "first" is used because the Haar wavelet is firstly developed among the various wavelet filters.

[18] Additional information regarding wavelet and scaling filters (including the Haar and longer compactly supported orthogonal wavelets) and their properties may be found in Gençay et al. (2002).

$$h_{2,i} = \{g * h'_i\} = \sum_{j=0}^{L-1} g_j h'_{i-j}, \quad \text{for } i = 0, \ldots, 3, \tag{A.5}$$ [19]

From this equation, we can obtain the scale 2 wavelet filter, $h_2 = (1/2, 1/2, -1/2, -1/2)$ with length $L_2 = 4$. The wavelet coefficient d_2 may be obtained using h_2 via $2d_2 = \{h_2 * R_t\}$ where the factor of 2 is a normalization constant. The scale 2 Haar wavelet filter first averages two pairs of returns and then proceeds to difference them. Thus, the wavelet coefficients d_2 are associated with changes on a scale of two. By defining $g' = (g_0, 0, g_1) = (1/\sqrt{2}, 0, 1/\sqrt{2})$, the scale 2 Haar scaling filter is obtained using:[20]

$$g_{2,i} = \{g * g'_i\} = \sum_{j=0}^{L-1} g_j g'_{i-j}, \quad \text{for} \quad i = 0, \ldots, 3, \tag{A.6}$$

so that $g_2 = (1/2,\ 1/2,\ 1/2,\ 1/2)$. It is observed that g_2 is a simple average of four consecutive returns. The scaling coefficients s_2 may be obtained directly using g_2 via $2s_2 = \{g_2 * R_t\}$.

Analogously, the scale 3 wavelet and scaling coefficients may be obtained by increasing numbers of zeros inserted between the wavelet and scaling filter coefficients such as h and g with length $L = 2$. For example, $h'' = (h_0, 0, 0, h_1) = (1/\sqrt{2}, 0, 0, -1/\sqrt{2})$ and define the scale Haar wavelet filter to be $h_3 = \{\{g * g'_i\} * h''_i\} = \{g_2 * h''_i\}$with the wavelet coefficients:

$$h_3 = \left(\frac{1}{\sqrt{8}}, \frac{1}{\sqrt{8}}, \frac{1}{\sqrt{8}}, \frac{1}{\sqrt{8}}, \frac{-1}{\sqrt{8}}, \frac{-1}{\sqrt{8}}, \frac{-1}{\sqrt{8}}, \frac{-1}{\sqrt{8}}\right)$$

with length $L_3 = 8$. Generally, the length of a scale j wavelet filter can be calculated by $L_j = (2^j - 1)(L - 1) + 1$. Similarly, the scale 3 scaling filter coefficients are calculated as $g_3 = \{\{g * g'_i\} * g''_i\} = \{g_2 * g''_i\}$where $g'' = (g_0, 0, 0, g_1) = (1/\sqrt{2}, 0, 0, 1/\sqrt{2})$, with coefficients:

[19] The series stop at 3. As can be seen above Eq. A.5 we define $h' = (h_0, 0, h_1) = (1/\sqrt{2}, 0, -1/\sqrt{2})$. If we fully expand the equation (A.5), it has the following form:

$$\text{When } i = 0,\ h_{2,0} = g_0 h'_0 + g_1 h'_{-1} = \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \times 0 = \frac{1}{2},$$

$$\text{When } i = 1,\ h_{2,1} = g_0 h'_1 + g_1 h'_0 = \frac{1}{\sqrt{2}} \times 0 + \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = \frac{1}{2}$$

$$\text{When } i = 2,\ h_{2,2} = g_0 h'_2 + g_1 h'_1 = \frac{1}{\sqrt{2}} \times -\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \times 0 = -\frac{1}{2}$$

$$\text{When } i = 3,\ h_{2,3} = g_0 h'_3 + g_1 h'_2 = \frac{1}{\sqrt{2}} \times 0 + \frac{1}{\sqrt{2}} \times -\frac{1}{\sqrt{2}} = -\frac{1}{2}$$

In the above calculation, h_{-1} and h_3 are considered as 0 because it is not defined. This happens when we generalize notation using summation.

[20] For other filters such as the Daubechies least asymmetric wavelet filter of length 8 (LA(8)), the construction of the higher scale filters are the same as in this example. For instance, the scale 1 LA(8) scaling filter, g, is (–0.0758, –0.0296, 0.4976, 0.8037, 0.2979, –0.0992, –0.0126, 0.0322). In this case, g' is defined as (–0.0758, 0, –0.0296, 0, 0.4976, 0, 0.8037, 0, 0.2979, 0, –0.0992, 0, –0.0126, 0, 0.0322). For the various scaling filter coefficients, see Gencay et al. (2002, pp. 114–11).

$$g_3 = \left(\frac{1}{\sqrt{8}}, \frac{1}{\sqrt{8}}, \frac{1}{\sqrt{8}}, \frac{1}{\sqrt{8}}, \frac{1}{\sqrt{8}}, \frac{1}{\sqrt{8}}, \frac{1}{\sqrt{8}}, \frac{1}{\sqrt{8}}\right)$$

Similar to the calculation of the scale 2 wavelet and scaling coefficients, the scale 3 wavelet and scaling coefficients are calculated via $\sqrt{8}d_3 = \{h_3 * R_t\}$ and $\sqrt{8}s_3 = \{g_3 * R_t\}$, respectively. This procedure may be repeated up to the scale $J = \log 2^T$ with the resulting wavelet and scaling coefficients organized into the vector $w = (d_1, d_2, \ldots, d_J, s_J)$.

The wavelet coefficients from scale $j = 1, 2, \ldots, J$ are associated with the frequency interval $[1/2^{j+1}, 1/2^j]$ while the remaining scaling coefficients s_J are associated with the remaining frequencies $[0, 1/2^{j+1}]$. In this specification, the wavelet coefficients $d_J, \ldots, d_1$, which can capture the higher frequency oscillations, represent increasingly fine scale deviations from the smooth trend. s_J represents the smooth coefficients that capture the trend.

Finally, using the wavelet coefficients, the wavelet variance for scale λ_j can be estimated by:

$$\sigma^2(\lambda_j) \equiv Var(d_{jt}) \tag{A.7}$$

References

Benson K, Faff R (2003) Performance of Australian international equity trusts. Int Finan Mark Inst Money 13:69–84

Bird R, Chin H, McCrae M (1983) The Performance of Australian superannuation funds. Aust J Manage 8:49–69

Chan J, Hung M, Lee C (2003) An Inter-temporal CAPM Approach to Evaluated Mutual Fund Performance. Rev Quant Finance Acc 20:415–433

Dacorogna M, Gençay R, Muller U, Olsen R, Pictet O (2001) An introduction to high-frequency finance. Academic Press, London, UK

Dowd K (1999) Financial risk management. Finan Analysts J 55:65–71

Ferson W, Schadt R (1996) Measuring fund strategy and performance in changing economic conditions. J Finan 51:425–462

Gallagher DR (2001) Attribution of investment performance: an analysis of Australian pooled superannuation funds. Account Finance 41:41–62

Gençay R, Selçuk F, Whitcher B (2002) An introduction to wavelets and other filtering methods in finance and economics. London, Academic Press

Gençay R, Selçuk F, Whitcher B (2003) Systematic Risk and Time Scales. Quant Finance 3:108–116

Hallahan T, Faff R (1999) An examination of australian equity trusts for selectivity and market timing performance. J Multinational Finan Manage 9:387–402

Henriksson R, Merton R (1981) On market timing and investment performance II: statistical procedure for evaluating forecasting skills. Journal of Business 54:513–533

Hodges CW, Taylor WR, Yoder JA (1997) Stocks, bonds, the Sharpe ratio, and the investment horizon. Finan Analysts J 53:74–80

Holmes K, Faff R (2004) Stability, asymmetry and seasonality of fund performance: an analysis of australian multi-sector managed funds. J Bus Finan Acc 31:539–578

In F, Kim S (2006) The hedge ratio and the empirical relationship between the stock and futures markets: a new approach using wavelet analysis. J Bus 79:799–820

Jobson J, Korkie B (1981) Performance hypothesis testing with Sharpe and treynor measures. J Finance 36:889–908

Kazemi H, Mahdavi M, Schneeweis T (2003) Generalized Sharpe ratio: a defense against Sharpened Sharpe ratios. CISDM Working Paper, University of Massachusetts

Kim S, In F (2005a) Multihorizon Sharpe ratio. J Portfol Manage 31:105–119

Kim S, In F (2005b) Stock returns and inflation: new evidence from wavelet analysis. J Empirical Finance 12:435–444

Knight J, Satchell S (2005) A re-examination of shape's ratio for log-normal prices. Appl Math Finance 12:87–100

Lee C, Rahman S (1990) Market timing and mutual fund performance: an empirical investigation. J Bus 63:261–278

Leland HE (1999) Beyond mean-variance: performance measurement in a nonsymmetrical world. Finan Analysts J 55:27–36

Levy H (1972) Portfolio Performance and the Investment Horizon. Manage Sci 18:645–653

Lo AW (2002) The statistics of Sharpe ratios. Finan Analysts J 58:36–52

Lo AW (2003) The statistics of Sharpe ratios: authors response. Finan Analysts J 59 (November/December, 17)

Marisetty V, Ariff M (2003) Security selection and market timing of retail superannuation fund managers. Int J Finance 15:2489–2507

Modigliani F, Modigliani L (1997) Risk-adjusted performance. J Portfol Manage 23:45–54

Neilson MØ, Frederiksen P, Finite sample accuracy and choice of sampling frequency in integrated volatility estimation. J Empirical Finance (in press)

Nielson TL, Vassalou M (2004) Sharpe ratios and alphas in continuous time. J Finan Quant Anal 39:103–114

Owen J, Rabinovitch R (1999) Ranking portfolio performance by a joint means and variance equality test. J Appl Econ 2:97–130

Percival DB, Walden AT (2000) Wavelet methods for time series analysis. Cambridge University Press, UK

Ramsey J (2002) Wavelets in economics and finance: past and future, studies in non-linear dynamics and econometrics, vol 6(3), Berkeley Electronic Press, pp 1090–1112

Ramsey JB, Lampart C (1998a) Decomposition of economic relationships by timescale using wavelets. Macroecon Dynam 2:49–71

Ramsey JB, Lampart C (1998) The decomposition of economic relationships by time scale using wavelets: expenditure and income. Stud Nonlinear Dynam Econometrics 3:23–42

Sawicki J, Ong F (2003) Evaluating managed fund performance using conditional measures. Pacific Basin Finance J 8:505–528

Sharpe W (1966) Mutual fund performance. J Bus 34:119–138

Sharpe W (1994) The Sharpe ratio. J Portfol Manage 21:49–58

Siegel JJ (1998) Stocks for the long run. McGraw-Hill, New York

Sinclair N (1990) Market timing ability of pooled superannuation funds. Acc Finance 30:511–565

Strang G, Nguyen T (1996) Wavelets and filter banks. Wellesley-Cambridge Press, Wellesley, MA

Stutzer M (2000) A portfolio performance index. Finan Analysts J 56:52–61

Treynor J, Mazuy K (1966) Can mutual funds outguess the market? Harvard Bus Rev 44:131–136

Wolf M (2003) The statistics of Sharpe ratios: a comment. Finan Analysts J 59 (November/December, 17)

Ziemba W (2005) The symmetric downside-risk Sharpe ratio. J Portfol Manage 108–122